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Small Signal Modelling of a Buck Converter using State Space Averaging for Magnet Load

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***Abstract*: Nowadays, step-down power converters such as buck scheme are widely employed in a variety of applications such as power supplies, spacecraft power systems, hybrid vehicles and power supplies in particle accelerators. This paper presents a comprehensive small-signal model for the DC-DC buck converter operated under Continuous Conduction Mode (CCM) for a magnetic load. Initially, the buck converter is modeled using state-space average model and dynamic equations, depicting the converter, are derived. The proposed model can be used to design powerful, precise and robust closed loop controller that can satisfy stability and performance conditions of the DC-DC buck regulator. This model can be used in any DC-DC converter (Buck, Boost, and Buck-Boost) by modifying the converter mathematical equations.**

Keywords: **Small-signal model, State-space averaging; Buck Converter, Magnet Load.**

I. INTRODUCTION

The ever expanding demand for smaller size, portable, and lighter weight with high performance DC-DC power converters for industrial, communications, residential, and aerospace applications is currently a topic of widespread interest [1]. Switched-mode DC-DC converters have become commonplace in such integrated circuits due to their ability to up/down the voltage of a battery coupled with high efficiency. The three essential configurations for this kind of power converters are buck, boost and buck–boost circuits, which provide low/high voltage and current ratings for loads at constant switching frequency [1]. The topology of DC-DC converters consists of linear (resistor, inductor and capacitor) and nonlinear (diode and dynamic switch) parts. A buck converter, as shown in Fig. 1, is one of the most widely recognized DC-DC converter. Magnet power supplies have some special characteristics than regular power supplies used for general purpose. These are used to feed electromagnets [2]-[7]. The strength and quality of the magnetic field produced by the electromagnet depends on the current passing through it. Hence magnet power supplies are current regulated. To model a magnet load a resistance in series with an inductor can be used. Fig. 2 shows a DC-DC buck converter with a magnet load. Because of the switching properties of the power devices, the operation of these DC-DC converters varies by time.

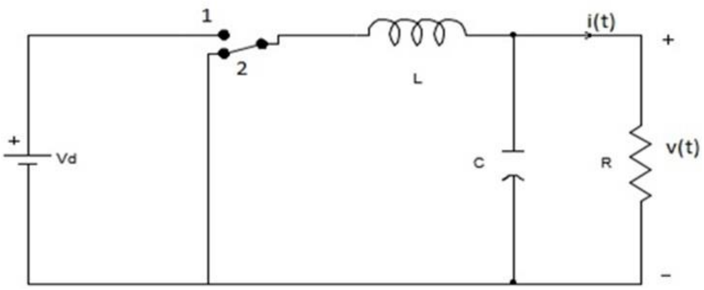

Figure.1: Basic DC-DC buck converter

Since these converters are nonlinear and time variant, to design a robust controller, a small-signal linearized model of the DC-DC converter needs to be found.

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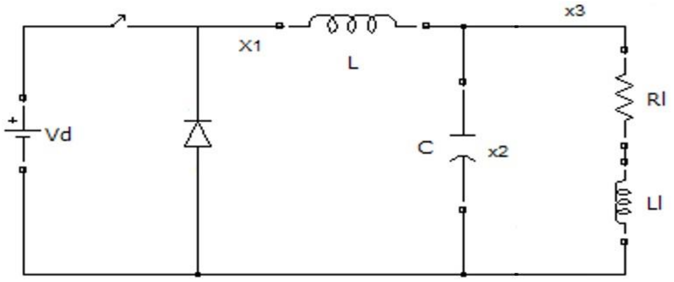

Figure 2: Buck converter with magnet load

Modelling of a system may be described as a process of formulating a mathematical description of the system. It entails the establishment of a mathematical input-output model which best approximates the physical reality of a system. Switching converters are nonlinear systems. For analysis of such converters the system needs to be linearized, hence the model of the switching converter is linearized. An advantage of such linearized model is that for constant duty cycle, it is time invariant. There is no switching or switching ripple to manage, and only the DC components of the waveforms are modelled. This is achieved by perturbing and linearizing the average model about a quiescent operating point [8], [9]. Various AC converter modelling techniques, to obtain a linear continuous time-invariant model of a DC-DC converter, have appeared in the literature. Nevertheless, almost all modelling methods, including the most prominent one, state-space averaging [10] will result in a multi-variable system with state-space equations, ideal or non-ideal, linear or nonlinear, for steady state or dynamic purpose. The proposed method is based on Laplace transforms. The simulation results in MATLAB software was used to confirm the validity of the hypothetical investigation. This paper develops a model that describes the AC small-signal linear time-invariant circuit model of a DC-DC buck converter for magnet load. The proposed model can be used to design a precise and robust closed loop controller**.** Section II presents state-space average methodology used, steps of power stage modelling and steady state solution. Perturbation and linearization about a quiescent operating point is also applied. Finally, the canonical form is illustrated. In section III, simulation of the proposed model for hypothetical system parameters is shown. In Section IV, the conclusions and future work are discussed.

II. SMALL SIGNAL MODELLING

To design the control system of a converter, it is necessary to model the behaviour of dynamic converter. Unfortunately, understanding of converter dynamic behaviour is hampered by the nonlinear time-varying nature of the switching and pulse width modulation process. Power stage modelling for DC-DC buck converter with magnet load based on state-space average method can be achieved to obtain an accurate mathematical model of the converter. A sate-space averaging methodology is a mainstay of modern control theory and most widely used to model DC-DC converters. The state space averaging method use the state-space description of dynamical systems to derive the small-signal averaged equations of PWM switching converters. The state space dynamics description of each time-invariant system is obtained. These descriptions are then averaged with respect to their duration in the switching period providing an average model in which the time variance is removed, which valid for the entire switching cycle. The resultant averaged model is nonlinear and time-invariant. This model is linearized at the operating point to obtain a small signal model. The linearization process produces a linear time invariant small-signal model. Finally, the time-domain small signal model is converted into a frequency-domain, or *s-domain*, small-signal model, which provides transfer functions of power stage dynamics. The resulting transfer functions embrace all the standard s-domain analysis techniques and reveal the frequency-domain small-signal dynamics of power stage. In the method of state-space averaging, an exact state-space description of the power stage is initially formulated. The resulting state-space description is called the switched state space model. The power stage dynamics during an ON-time period can be expressed in the form of a state-space equation as [11], [12]:

$$dx/dt = A_1 x + B_1 v_d \quad (1)$$

$$dx/dt = A_2 x + B_2 v_d \quad (2)$$

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Where, $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ are state matrix and $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ are vectors. The output voltage v_o is described as

$$v_o = C_1 x \quad (3)$$

$$v_o = C_2 x \quad (4)$$

Equations (1) and (3) are the state equation of the circuit when the switch is ON, whereas, equations (2) and (4) are the state equations when the switch is OFF. Averaging the State-Variable description using the duty ratio *d* we get

$$dx/dt = [A_1 d + A_2(1-d)]x + [B_1 d + B_2(1-d)]v_d \quad (5)$$

$$v_o = [C_1 d + C_2(1-d)]x \quad (6)$$

Introducing small AC perturbation and separation of AC and DC components.

$$\dot{\tilde{x}} = Ax + Bv_d + A\tilde{x} + [(A_1 - A_2)x + (B_1 - B_2)v_d]\tilde{d} + \text{higher order terms.} \quad (7)$$

And

$$v_o + \tilde{v}_o = Cx + C\tilde{x} + [(C_1 - C_2)x]\tilde{d} \quad (8)$$

Where, $A=A_1d+A_2(1-d)$

$B= B_1d+B_2(1-d)$

$C= C_1d+C_2(1-d)$

Equations (7) may be separated into DC (steady state) terms, linear small signal terms and non-linear terms. For the purpose of deriving a small-signal AC model, the DC terms can be considered known constant quantities. It is desired to neglect the nonlinear AC terms, then each of the second-order nonlinear terms is much smaller in magnitude that one or more of the linear first-order AC terms. Also the DC terms on the right-hand side of the equation are equal to the DC terms on the left-hand side, or zero. So the desired small-signal linearized state-space equations are obtained as:

$$\dot{\tilde{x}} = A\tilde{x} + [(A_1 - A_2)x + (B_1 - B_2)v_d]\tilde{d} \quad (9)$$

$$\tilde{v}_o = C\tilde{x} + [(C_1 - C_2)x]\tilde{d} \quad (10)$$

Transformation of the AC Equation in to s-Domain by taking Laplace transform of equation (9) and (10) to obtain the Transfer Function

$$\dot{\tilde{x}}(s) = [sI - A]^{-1}.[(A_1 - A_2)x + (B_1 - B_2)v_d] \quad (11)$$

Substituting equation (11) in (10) we get the output to duty ratio transfer function as:

$$G_p(s) = C.[sI - A]^{-1}.[(A_1 - A_2)x + (B_1 - B_2)v_d] + [(C_1 - C_2)x] \quad (12)$$

As shown in Fig. 2 when switch is ON, Diode is reverse biased, the converter circuit of Fig. 3 is obtained. The power stage dynamics during an ON-time period can be expressed in the form of a state-space equation is given as:

$$V_d = -L\dot{x}_1 - x_2 \quad (13)$$

$$x_2 - x_3 R_l - \dot{x}_3 L_l = 0 \quad (14)$$

$$x_1 = \dot{x}_2 C + x_3 \qquad (15)$$

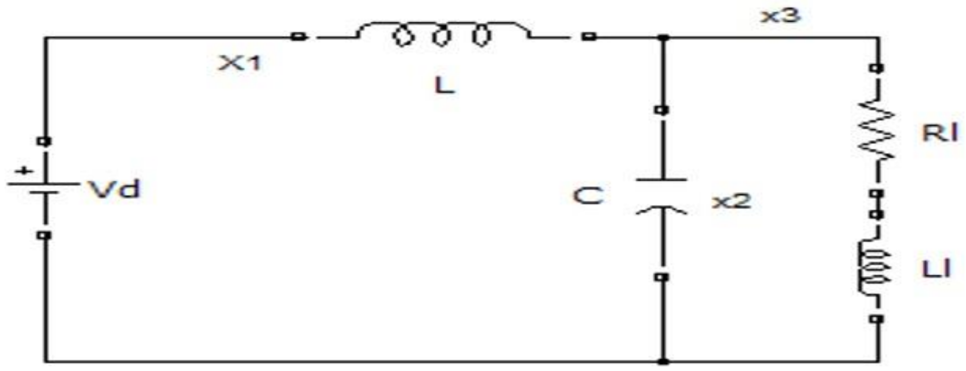

Figure 3: Buck converter equivalent circuit in ON state

Where, x_1 is the current through inductor L, x_2 is voltage across C, and x_3 is the current through the load.

Therefore, the state matrix for equation (13), (14) and (15) is given as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1/L & 0 \\ 1/C & 0 & -1/C \\ 0 & 1/L_l & -R_l/L_l \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix} V_d \qquad (16)$$

$$V_o = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (17)$$

Therefore,

$$A_1 = \begin{bmatrix} 0 & -1/L & 0 \\ 1/C & 0 & -1/C \\ 0 & 1/L_l & -R_l/L_l \end{bmatrix} \quad B_1 = \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

As shown in Fig. 2 when switch is OFF, Diode is forward biased, the converter circuit of Fig. 4 is obtained. The power stage dynamics during an OFF-time period can be expressed in the form of a state-space equation as

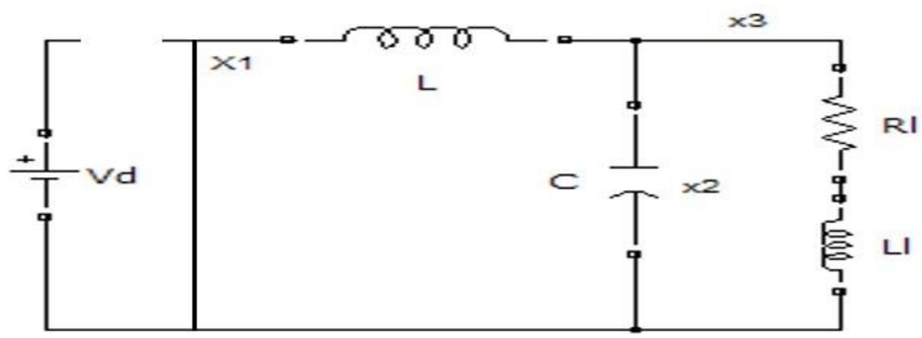

Figure 4: Buck converter equivalent circuit in OFF state

$$0 = L \dot{x}_1 + x_2 \qquad (18)$$

$$x_2 - x_3 R_l - \dot{x}_3 L_l = 0 \qquad (19)$$

$$x_1 = \dot{x}_2 C + x_3 \qquad (20)$$

Therefore, the state matrix for equation (18), (19) and (20) is given as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1/L & 0 \\ 1/C & 0 & -1/C \\ 0 & 1/L_l & -R_l/L_l \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} V_d \qquad (21)$$

$$V_o = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (22)$$

Therefore,

$$A_2 = \begin{bmatrix} 0 & -1/L & 0 \\ 1/C & 0 & -1/C \\ 0 & 1/L_l & -R_l/L_l \end{bmatrix} \quad B_2 = \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

From (16), (17), (21), (22) and (12) the transfer function can be obtained as:

$$G_p(s) = \frac{V_d (R_l + L_l s)}{CLL_l s^3 + CLR_l s^2 + (L + L_l)s + R_l} \qquad (23)$$

III. VALIDATION OF THE MODEL

In the previous section we have developed a small signal model of the buck converter for the magnet load. This model is exemplified with illustrative calculation in this section for a converter whose parameters are listed in table 1.

Table 1: Parameters of converter for illustrative calculations

Parameters	Values
V_d	30 V
L	30 mH
C	40 mF
L_l	50 mH, 100 mH, 500 mH
R_l	1 Ω

MATLAB is used for the calculation of the transfer function with the parameter given in table 1 and the following steps

- Calculate A_1 using (16)
- Calculate B_1 using (16)
- Calculate C_1 using (17)
- Calculate A_2 using (21)
- Calculate B_2 using (21)
- Calculate C_2 using (22)
- Calculate G_p using (12) and (23)

Fig. 5 is the frequency response of the converter transfer function G_p obtained from (23) for the values stated in table 1.

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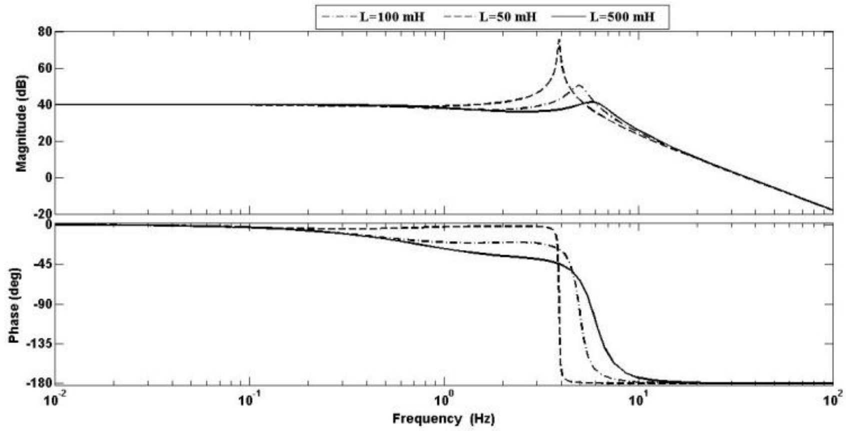

Figure 5: Frequency response of the converter transfer function for different magnet load

IV. RESULTS AND CONCLUSION

For lower value of magnet inductance, the resonance peak of the *LC* filter at the filter cut-off frequency has a very high amplitude, but as the value of the magnetic inductance is increased, the *LC* filter is damped. The frequency response of the transfer function of the converter was simulated for different values of magnet inductance i.e. 50 mH, 100 mH and 500 mH and from fig. 5 it can be observed that the damping provided by the load to the LC filter increases with the increase in the inductance value of the magnet load. Highly inductive magnets can provide extra damping to the filter as at higher frequencies high value of inductance will provide higher impedance.

We represented the small signal modelling and derived a general formula for the transfer function for DC-DC buck converter for different magnet load. The objective of our modelling efforts was to predict low frequency component and to observe how the converter responds for different magnet loads. To achieve this purpose, we applied the basic approximation of removing the high-frequency switching ripple by averaging over one switching period and then we derived transfer functions of buck converters for magnet load. The frequency response of the converter transfer function so obtained was then simulated for different values of magnet inductance.

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